Handling Exotic Positions For Counterparty Risk Management

Shahram Alavian*
Risk Analytics
Lehman Brothers International Europe†
25 Bank Street London

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Abstract

This paper proposes a simple approach for incorporating the difficult-to-value trades to the rest of counterparty portfolios using the existing and already-approved pricing models. It requires simulating only the risk factors that these pricers require for the population of Monte Carlo simulation paths. A smaller subset of paths can be used to generate a path of trade values using these pricers. An appropriate set of basis functions over the risk factors can be selected in order to model the conditional expectation values and use the model to map the conditional values to the population. However, it is assumed that the input and output values of the pricers are properly manipulated in order to create the appropriate path−dependency. The approach is applied to the case of in−the−money American put option where the counterparty risk exposure profiles are compared between the full valuation and the valuations using this method for various path numbers. The case is extended to a null value portfolio composed of two offsetting options where one is priced using the benchmark method and another using the proposed approach for various path numbers.

*For questions and comments please contact the author at shahram_alavian@yahoo.com.

†The opinions expressed are those of the author and do not necessarily reflect the views of author's employees, or any member of their staff.

1 Introduction

Counterparty risk groups (CRG) in financial brokers and dealers are constantly engaged in measuring, reporting, and improving the management of their counterparty risk exposures.

For over-the-counter (OTC) counterparties, these activities fundamentally involve evaluating their future exposure distributions created by aggregating the conditional values of each and every individual OTC trades within the portfolio.

In general, calculation of the exposure distribution, for each time horizon, involves simulation of all relevant market risk factors, for a large number of scenario-based paths, and valuation of each trade for each of those scenarios. The portfolio value distribution created by the variation of the market scenarios across all these scenarios is the building block of counterparty exposure reporting

Accuracy of these exposure distributions is crucial as they are used for variety of risk mitigation purposes; being calculation of the regulatory, as well as economic capital, or reporting any other risk measures such as expected exposure (EE) and potential future exposure (PFE), among host of others.

One of the many challenges of CRG is the ability of evaluating all trades, within a portfolio, since the lack of valuation for even a single trade can put the risk measures obtained for the entire portfolio in question. There are usually numerous reasons why CRG is not able to evaluate a trade. One reason is that valuation models for every single trade types are not available to the group. Another reason would be due to the limitations within the data feed. For example, a trade may possibly miss a maturity date or it might be represented in a proprietary script language un-translatable by CRG. In these instances, since the risk is unclear, the trade is commonly assigned a conservative exposure value to make sure its contribution to the portfolio is always risk incrementing. Lack of proper information causes these "risk incrementing" trades to significantly increase the capital requirements and saturate the counterparty's credit limit.

Theoretically, one can use the existing pricing and approved front office (FO) modules used within the firm in order to obtain the evaluations. However, in reality, there are two main problems: Trade aging and the time required in order to re-evaluate the trade for all number of paths and time horizons. Trade aging is the path dependency treatment of a trade as it is simulated in to the future. For example, in the context of trade aging, value of the cash

settled American option, which has been exercised, should be 0. Another challenge of using these pricing models is the time it takes to price the trade particularly the exotic ones. The objective of this paper is to address this issue by modeling the conditional values using FO pricers on a smaller, and more achievable, number of paths. The model can then be used to regenerate the conditional values for the whole population of paths. The method is crude and generates no more information than what pricing the trade over the same small number of paths would generate. However, it does provide a few advantages: the mark-to-market values are approved since they match, by construction, the firm's book of records. It frees CRG from duplication effort may it be the implementation of equivalent pricers or regenerating trade feeds. Furthermore, the approach would provide the risk dynamics and reports an appropriate contribution to the total risk of the portfolio as a function of number of sample paths; that is, the precision now is more matter of a scale and less of the methodology.
The remainder of this paper is organized as follows: Section 2 presents the approach and the assumptions therein. Section 3 addresses the issue of the trade aging. A step-by-step procedure of the approach is given in section 4 and the the approach is applied to the case of an American put option in section 5. Finally, some concluding remarks are provided in section 6.

2 Setup

Starting from t_j-forward measure, under the Monte Carlo simulation,

$$\mathbb{E}^j \left[B(t_j, t_k) x(t_k) \middle| \mathfrak{F}_j \right] = V_{ij} \tag{1}$$

where the indices i and j correspond to path i and the t_j, respectively. Since the conditional trade value is an element of the L^2 space of square$-$ integrable functions, it can be represented as a linear combination of a countable set of $\mathfrak{F}-$measurable basis functions [1]. That is, in a Markovian space, one can represent the above equation as

$$V_{ij} = \sum_{k=0}^{\infty} b_j^k f_{ij}^k \tag{2}$$

where

$$f_{ij}^k \equiv f_{ij}^k \left[\vec{\eta} \left(\mathfrak{F} \right) \right] \tag{3}$$

represents some chosen orthonormal basis function that depends on a set of $\mathfrak{F}-$measurable market risk factors $\overrightarrow{\eta}$ driving the price of the trade. The factors b_j^k are the coefficients to be calculated. The first consequence of Markovian assumption is that path-dependency of the trade values is altered. This is easy to see since V_{ij} should in directly reality depend on the filtration $\mathfrak{F}$ while the dependency of the right hand side of (2) is through the basis functions. Practically speaking, three more simplifications are commonly made. One is the choice of the basis function. Second is the number of terms in the series and the third is the dependencies of the basis function on the market risk factors. Whatever the choice, assuming that they are within the acceptable tolerance,

$$V_{ij} \simeq B_j^0 + \sum_{k=1}^{N} B_j^k F_{ij}^k \tag{4}$$

$$= V\left(\vec{B_j^k} \middle| \vec{F_j}\right) \tag{5}$$

where each expansion is now approximated to N terms of new simplified functions, F. In matrix form the above equation can be written as

$$[\mathbf{V}_{ij}]_{M\times 1} = [\mathbf{F}_j]_{M\times N} \left[\mathbf{B}_j\right]_{N\times 1} \tag{6}$$

where M is the number of the Monte Carlo paths. In the above equation the first column of the matrix $[\mathbf{F}_j]_{M\times N}$ representing the basis functions is 1 for all rows and the first element of $\left[\mathbf{B}_j\right]_{N\times 1} = B_j^0$. Given the vector $\vec{F_j}$ for each time t_j and the functional form of (6) one can recover the path$-$independent values conditional on each set of $\vec{F_j}$. Obviously, the best situation is when this is done for the whole population of M paths. In reality, this is impractical since the existing pricing modules cannot usually be used for generating the complete conditional trade value distribution due to their consumption of IT resources. Therefore, the next best approach maybe to evaluate the trade, at each time t_j, on a set of $m \leq M$ sample paths and use (5) to estimate the complete distribution on the population of M paths.
Consider a subset of m paths ($m <= M$) large enough to provide a adequate portfolio value distribution

$$\left[\tilde{\mathbf{V}}_{ij}\right]_{m\times 1} = \left[\tilde{\mathbf{F}}_j\right]_{m\times N} \left[\tilde{\mathbf{B}}_j\right]_{N\times 1} \tag{7}$$

It is, therefore, appropriate to assume that within the accepted tolerance

$$\left[\tilde{\mathbf{B}}_j\right]_{N\times 1} \simeq \left[\mathbf{B}_j\right]_{N\times 1} \tag{8}$$

Since

$$\left[\tilde{\mathbf{B}}_j\right]_{N\times 1} = \left[\left[\tilde{\mathbf{F}}_j\right]_{N\times m}\left[\tilde{\mathbf{F}}_j\right]_{m\times N}\right]^{-1}\left[\tilde{\mathbf{F}}_j\right]_{N\times m}\left[\tilde{\mathbf{V}}_{ij}\right]_{m\times 1} \tag{9}$$

and considering (8), equation (6) can now be written as

$$\left[\mathbf{V}_{ij}\right]_{M\times 1} = \left[\mathbf{F}_j\right]_{M\times N}\left[\left[\tilde{\mathbf{F}}_j\right]_{N\times m}\left[\tilde{\mathbf{F}}_j\right]_{m\times N}\right]^{-1}\left[\tilde{\mathbf{F}}_j\right]_{N\times m}\left[\tilde{\mathbf{V}}_{ij}\right]_{m\times 1} \tag{10}$$

For each time horizon t_j, the above equation maps the conditional values from m on to M paths. Obviously, the fixed assumptions are the replacing the Markovian space with the existing one and (6). Assumption (8) are simply a question of scale for m since for $m = M$ the assumption (8) disappear. As previuosly noted, this approach does not provide any more information that what the simulation of the subset of m paths would do. However, the motivation for this paper is to create an appropriate risk contribution as a function of scalable path number m.

3 Trade Aging

In this approach, trade aging should be a part of the pricing module. This is essential in order to ensure an appropriate path dependency. This an important aspect of portfolio evaluation for counterparty risk exposure. As an example, even a simple vanilla swap would need to *estimate* the reset rate of the first upcoming floating coupon if the valuation time is passed the reset time. Since this estimation is not a simulated market input values the pricer would need to make this estimation. As a rule of thumb, when a trade type becomes more "exotic", the dependency to the path can increase to a point that without the appropriate set of historical values, the trade can no longer be evaluated. Such path dependency is commonly ensured by implementing a "wrapper" module around the existing pricer to ensure the appropriate seasoning of the trade as much as possible. This wrapper, in general, has two responsibilities: 1) To provide a path dependent input values to the pricer such as averages, reset rates, etc. 2) To override the behavior of the trade by manipulating the input/output values. For example, it is the responsibility of the wrapper to remember that the options has been exercised/knocked-in/knocked-out, etc., and it would be this very wrapper that would modify the strike of a physically settled swaption in order to obtain the underlying swap.

4 Step by Step

Almost in all CRG projects, the main issues are related to elements outside CRG's control such as upstream systems, feed quality or lack of time series and appropriate information. Therefore, due to these firm-specific issues there could not be a single global step-by-step recipes. These steps, therefore, are to provide a sense of stylized application of the approach rather than a universal solution.

- Step 1: Simulate M paths of correlated market risk factors[1] across J time horizons.
- Step 2: Select $m \leq M$ paths
- Step 3: Select the set of L risk factors needed for pricing a trade under a given model. This means that for each path there are L risk factors at each of J time horizons. This would constitute a batch of hyper cube of m paths of L risk factors across J time horizons.
- Step 4: For each of the m paths evaluate the trade across all J time horizons. [For the physically settled options, the values of the underlying for the same set of the risk factors can be obtained using the same procedure given here].
- Step 5: For each of the time horizons t_j export the path-dependent value of the trade *and* whether the trade has been canceled. Canceled paths would be excluded from (7). Note that this very path could have been included in the previous time horizons.

[1] By "market risk factors" it is meant the market risk factors needed by the pricing modules in order to price a trade. Their simulation could follow either the historical or market implied measure.

- Step 6: Results of each valuation should be a matrix $m \times J$ of valuations and a matrix of $m \times J$ markers defining the state at which the trade was canceled[2].

- Step 7: Start from $j = 1$. Note that for $j = 0$ one pricing is sufficient for all populations paths.

- Step 8: Select the paths where the trade has not been expired. This would be $m' \leq m$ paths.

- Step 9: Use m' in (10) to obtain the matrix of conditional values $[\mathbf{V}_{ij}]_{M\times 1}$.

- Step 10: Increment j by one: $j = j + 1$. if $j <= J$ go to step 8.

- Step 11: [Path dependency improvement] Once the options and their underlying conditional values are obtained, the path−dependency modification, for all M paths, can be more conveniently achieved for exercise, knock-in(out), etc.[3].

5 An Example

In this section the steps above are applied to the case of an in−the−money American Put Option with maturity of 9 years and a strike of 1.80. With this strike, 40% of paths have been exercised *before* the maturity. This is stylized case to some−how reflect a real aged trade. The option is evaluated using the population of $1,000$ paths (Benchmark). However, it is also evaluated, by the same pricer, using (10) for different number of paths (Partial): 50, 100, and $1,000$. To single-out the effect of the proposed approach a null portfolio is also created using two offsetting options. By construction, this portfolio should have no risk, by construction, and any residual risk would

[2]Note that trade value of exactly 0 reflects the fact that the trade has been canceled and non-existent. Therefore, the same valuation matrix can also be used for selecting the path at which the trade has been expired/canceled.

[3] for example for the case of the option, for each of the M paths go through the path from $j = 0$, compare the price of the option and its underlying. If the option is to be exercised, replace the option with the underlying value for that time horizon and replace the remaining values of the path by 0 (cash settled options) or the underlying (for physical settled)

be fictitious and based on the approximations made through-out the process. The proposed model[4] for the conditional value is

$$\begin{aligned} V_{ij} &= B_j^0 + \sum_{k=1}^{j-1} B_j^k V_{ik} + \sum_{k=j}^{N} B_j^k F_{ij}^k \\ &= B_j^0 + B_j^1 V_{i1} + B_j^2 V_{i2} + \cdots + B_j^{j-1} V_{ij-1} + B_j^j S_{ij} + B_j^{j+1} \sigma_{ij} \end{aligned} \tag{11}$$

To be able to reconstruct the results more details of the setup is provided. The stock follows the log−normal process of

$$\ln S(t+dt) = \ln S(t) + \left(0.10 - 0.50 * .25^2\right) dt + 0.25 dw_s(t) \tag{12}$$

with $S(0) = 1.00$. The implied volatility was chosen to follow

$$\ln \sigma(t+dt) = \left[-0.01 + 0.20 * \ln \sigma(t)\right] dt + 0.50 dw_\sigma(t) \tag{13}$$

with $\sigma(0) = 0.30$, $E[dw_s dw_\sigma] = -0.7dt$ and $dt = 0.50$. The option was priced using [2]. Figure 1(a) represents the comparison of EE profiles between the Benchmark and the Partial pricing for different number of paths. Figure 1(b) compares the EE profiles of the null portfolio for the three path numbers. Figure 2 illustrates the same comparison for the 95 percentile PFE profiles. It is important to note that the point here is the comparative advantage of this method over the crude and over estimation of the risk through some simplistic approach. Comparatively, the EE profile of the null portfolio has dropped to 10%. Every thing else being the same, under BASEL II Internal Model Method (IMM), the 1-year EE profile is the drives the regulatory capital. Regardless of the firm's methodology for portfolios containing "risk incrementing" trades, it is clear that the drop of the profile 1(a) to less than 5% is a significant improvement. Moreover, it is still important to note that the PFE profile 2(b) is still below the PFE profile of a single trade.

[4] Other models such as

$$\begin{aligned} V_{ij} &= B_j^0 + \sum_{k=1}^{j-1} B_j^k S_{ik} + \sum_{k=j}^{N} B_j^k F_{ij}^k \\ &= B_j^0 + B_j^1 Si1 + B_j^2 S_{i2} + \cdots + B_j^{j-1} S_{i(j-1)} + B_j^j S_{ij} + B_j^{j+1} \sigma_{ij} \end{aligned}$$

have also been tried. The results varied very little. However, (11) gives a more generic representation.

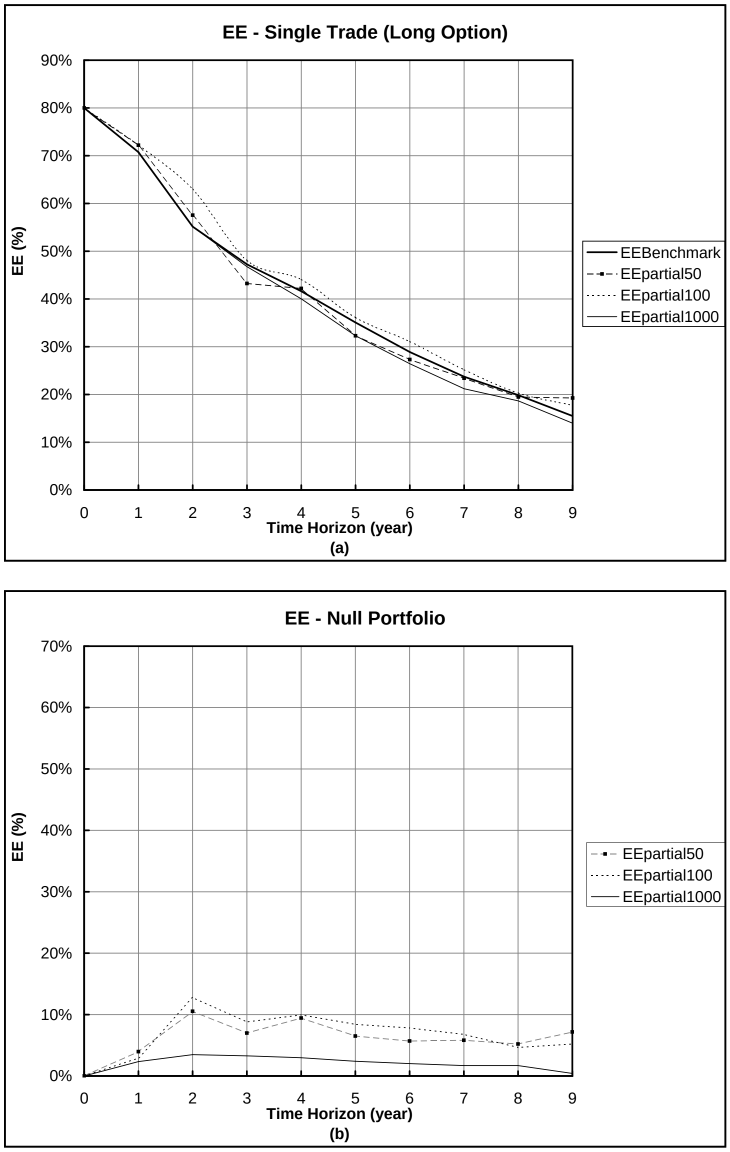

Figure 1: (a) Comparison of expected exposure profiles between full valuation (Benchmark) and using (10) from a sample set (Partial) of 50, 100 and $1,000$ paths for a single American put option. (b) Expected exposure profiles of a null portfolio composed of two offsetting American options where one trade is priced along each of the $1,000$ and another is priced using (10) from a sample set (Partial) of 50, 100 and $1,000$ paths.

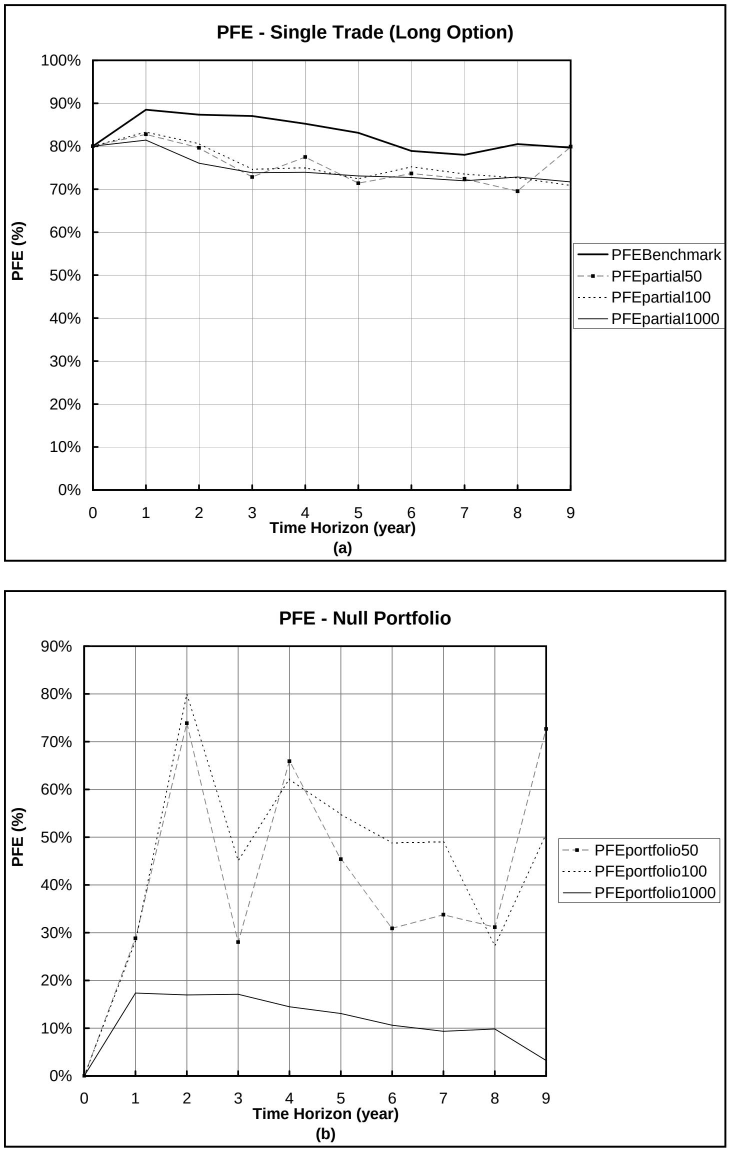

Figure 2: (a) Comparison of potential future exposure profiles between full valuation (Benchmark) and using (10) from a sample set (Partial) of 50, 100 and $1,000$ paths for a single American put option. (b) Potential future exposure profiles of a null portfolio composed of two offsetting American options where one trade is priced along each of the $1,000$ and another is priced using (10) from a sample set (Partial) of 50, 100 and $1,000$ paths.

6 Concluding Remarks

Using a subset of simulated paths of risk factors a collection of valuation paths can be obtained. The values can then be modeled as a function of a set of basis functions. Though, the results are affected by the approximations, they do present a degree of risk dynamics and more appropriate representation of their contribution to the total risk. The feature of the approach is that most of the work is done *around* the existing pricing models and there is less need to re-implement the models themselves.

References

[1] Francis A. Longstaff, and Eduardo S. Schwartz, 2001, "Valuing American Options by Simulation: A Simple Least Squares Approach", *The Review of Financial Studies*, 2001-14, **113-147**.

[2] Giovanni Barone-Adesi and Robert E. Whaley, "Efficient Analytical Approximation of American Option Value", *Journal of Finance*, 1987-2, **301-320**